Quiz 3 (10pts)
Math 214 Section Q1 Winter 2010

Your name:________________________________ ID#:______________

Please, use the reverse side if needed.

1.(5 pts) Test the series for absolute convergence, conditional convergence or divergence

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n+5}.$$

Solution.

First, let us test the series for absolute convergence. Consider

$$\sum_{n=1}^{\infty}\left|(-1)^{n+1}\frac{1}{n+5}\right| = \sum_{n=1}^{\infty}\frac{1}{n+5} = \frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\cdots .$$

This is the harmonic series without the first 5 terms. Thus, divergent.

This means that the original series is not absolutely convergent.

To test the original series for convergence, we will use the alternating series test. Let $u_n = \frac{1}{n+5}$.

i) $\lim_{n\to\infty}\frac{1}{n+5} = 0$,

ii) $u_n = \frac{1}{n+5} > \frac{1}{n+6} = u_{n+1}$.

Since the sequence u_n is decreasing and its limit is zero, the alternating series test yields that the original series is convergent. So, the series is convergent, but not absolutely convergent. Thus, it is **conditionally convergent**.

2.(5 pts) Find the radius of convergence and interval of convergence of the power series

$$\sum_{n=1}^{\infty}\frac{(x-2)^n}{n4^n}$$

Solution.

We will use the ratio test.

$$\lim_{n\to\infty} \frac{|x-2|^{n+1}}{(n+1)4^{n+1}} \frac{n4^n}{|x-2|^n} = \lim_{n\to\infty} \frac{|x-2|}{4} \frac{n}{(n+1)} = \frac{|x-2|}{4}$$

If the this limit is less than 1, then the series is convergent.

$$\frac{|x-2|}{4} < 1,$$

$$|x-2| < 4,$$

$$-4 < x-2 < 4,$$

$$-2 < x < 6.$$

Thus the radius of convergence is $R = 4$.

We also need to test the endpoints of the interval $(-2, 6)$.

If $x = -2$, then the series becomes

$$\sum_{n=1}^{\infty} \frac{(-4)^n}{n4^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}.$$

This is the alternating harmonic series. It is convergent by the alternating series test.

If $x = 6$, the the series becomes

$$\sum_{n=1}^{\infty} \frac{4^n}{n4^n} = \sum_{n=1}^{\infty} \frac{1}{n}.$$

This is the harmonic series. Divergent.

Therefore, the interval of convergence of the power series is $[-2, 6)$ (i.e. $-2 \le x < 6$).

Answer: $R = 4$**,** $I = [-2, 6)$**.**